The Joy of Probabilistic Answer Set Programming

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Goal: to show that the credal semantics for Probabilistic Answer Set Programming (PASP) leads to a very useful modeling language.

Answer set programming (ASP)

A program is a set of rules such as

\[
green(X) \lor green(X) \lor blue(X) :\neg node(X), \text{not barred}(X).
\]

A fact is a rule with no subgoals: node(1).

Stable model semantics:
- Herbrand base: all groundings generated by constants in the program.
- Minimal model is a model (interpretation that satisfies all rules) such that none of its subsets is a model.
- Answer set: a minimal model of the reduct (propositional program obtained by grounding, then removing rules with not, then removing negated subgoals).

Probabilistic ASP (PASP)

A PASP program contains rules, facts, and probabilistic facts such as

\[
0.25 :: \text{edge}(\text{node}1, \text{node}2).
\]

A total choice induces an Answer Set Program.

Acyclic propositional (Bayesian network):

\[
\begin{align*}
0.01 &:: \text{trip}. \\
0.5 &:: \text{smoking}. \\
\text{tuberculosis} &:: \text{trip}, a1. \\
\text{tuberculosis} &:: \text{not} \text{trip}, a2. \\
0.05 &:: a1. \quad 0.01 &:: a2. \\
\text{cancer} &:: \text{smoking}, a3. \\
\text{cancer} &:: \text{not} \text{smoking}, a4. \\
0.1 &:: a3. \quad 0.01 &:: a4. \\
\text{either} &:: \text{tuberculosis}. \\
\text{either} &:: \text{cancer}. \\
\text{test} &:: \text{either}, a5. \quad 0.98 &:: a5. \\
\text{test} &:: \text{either}, a6. \quad 0.05 &:: a6.
\end{align*}
\]

Acyclic relational:

\[
\begin{align*}
\text{apt}(X) &:: \text{student}(X), a1. \quad 0.7 &:: a1. \\
\text{easy}(Y) &:: \text{course}(X), a2. \quad 0.4 &:: a2. \\
\text{pass}(X,Y) &:: \text{student}(X), \text{apt}(X), \text{course}(Y), \text{easy}(Y). \\
\text{pass}(X,Y) &:: \text{student}(X), \text{apt}(X), \text{course}(Y), \text{not} \text{easy}(Y), a3. \quad 0.8 &:: a3. \\
\end{align*}
\]

Closing...

In the paper: algorithm to compute lower/upper probabilities (future: better algorithms...).

In short: PASP with credal semantics is a very powerful language.

We can compute probabilities with some implicit quantification.

Stratified programs

\[
\begin{align*}
\text{edge}(X,Y) &:: \text{edge}(Y, X). \\
\text{path}(X,Y) &:: \text{edge}(X,Y). \\
\text{path}(X,Y) &:: \text{edge}(X,Z), \text{path}(Z,Y). \\
0.6 &:: \text{edge}(1,2). \\
0.1 &:: \text{edge}(1,3). \\
0.4 &:: \text{edge}(2,5). \\
0.3 &:: \text{edge}(2,6). \\
0.3 &:: \text{edge}(3,4). \\
0.8 &:: \text{edge}(4,5). \\
0.2 &:: \text{edge}(5,6).
\end{align*}
\]

PASP: Credal semantics

A total choice may induce a program with many answer sets.

Probability of each total choice may be distributed freely over answer sets: semantics is a credal set that dominates a two-monotone capacity.

Three-coloring:

\[
\begin{align*}
\text{red}(X) &:: \text{green}(X) \lor \text{blue}(X) :\text{node}(X). \\
\text{edge}(X,Y) &:: \text{edge}(Y, X). \\
\text{not}\text{colorable} &:: \text{edge}(X,Y), \text{red}(X), \text{red}(Y). \\
\text{not}\text{colorable} &:: \text{edge}(X,Y), \text{green}(X), \text{green}(Y). \\
\text{not}\text{colorable} &:: \text{edge}(X,Y), \text{blue}(X), \text{blue}(Y). \\
\text{red}(X) &:: \text{not}\text{colorable}, \text{node}(X), \text{not}\text{red}(X), \text{green}(X). \\
\text{blue}(X) &:: \text{not}\text{colorable}, \text{node}(X), \text{not}\text{blue}(X).
\end{align*}
\]

Then: \(P(\text{colorable}, \text{blue}(3)) = 0.976\).

Lower/upper probabilities: sharp probabilities with respect to appropriate questions: “What is the probability that I will be able to select a three-ordering where node 2 is red?” — answer is \(P(\text{colorable}, \text{red}(2))\).

This work was partially supported by CNPq and FAPESP.