Bayesian Decisions using Regions of Practical Equivalence (ROPE) and Imprecise Loss Functions

Patrick Schwaferts - Thomas Augustin

Institut für Statistik, Ludwig-Maximilians Universität München (LMU), Munich, Germany

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1. An Applied Bayesian Decision Problem

A decision between two actions $a_0$ and $a_1$ is of interest, i.e. $A = \{a_0, a_1\}$. The observed data $x$ are modeled parametrically with parameter $\theta \in \Theta$. The prior density $\pi(\theta)$ gets updated to the posterior density $\pi(\theta|x)$.

2. The Ideal Bayesian Solution

With the loss function $L : \Theta \times A \rightarrow \mathbb{R}_0^+$, the expected posterior loss

$$\rho(a) = \int L(\theta, a) \pi(\theta|x) \, d\theta \quad \text{for } a \in A$$

can be minimized to obtain the optimal action $a^* = \arg\min_a \rho(a)$.

3. Problem

Typically, the loss function $L$ is inaccessible.

4. Region of Practical Equivalence (ROPE)

The action $a_0$ is in accordance with a parameter null value $\theta^* \in \Theta$.

A ROPE for $\theta^*$ is a “range of parameter values that are equivalent to the null value for practical purposes” [Kruschke 2018, p. 272].

Its limits “depend on the practical purpose” [Kruschke 2015, p. 338] and need to be specified reasonably.

Denote the ROPE as $\Theta_0$ and $\Theta_1 = \Theta \setminus \Theta_0$.

The actions $a_0$ and $a_1$ are in accordance with $\Theta_0$ and $\Theta_1$, respectively.

5. ROPE in Decision Theory

Practical equivalence as constant loss function:

$$\forall \theta_0, \theta_1 \in \Theta_0; \forall a \in A: \, L(\theta_0, a) = L(\theta_1, a)$$

The ROPE-based Solution

With the ratio of expected posterior losses

$$r(K) = \frac{\rho(a_1)}{\rho(a_0)} = k \frac{p(\theta \in \Theta_0 | x)}{p(\theta \in \Theta_1 | x)}$$

the optimal action is

$$a^*(K) = \begin{cases} a_0 & \text{if } r(K) > 1 \\ a_1 & \text{if } r(K) < 1 \end{cases}$$

6. Define $k = k_1 / k_0$.

7. Problem

Typically, $k$ cannot be specified unambiguously. Any precise value for $k$ would be arbitrary.

8. Framework of Imprecise Probabilities

There is more to uncertainty than can be captured by precise probability values.

Sets of probabilities are employed as imprecise probabilities instead of precise probability values.

Imprecise probabilities are treated as an entity of its own.

9. An Imprecise Loss

The loss function $L$ is specified imprecisely by $K = \{K, \bar{K}\}$ instead of a precise $k$.

The precise values $K$ and $\bar{K}$ are bounds for reasonable choices of $k$.

10. The Imprecise ROPE-based Solution

Choose the action that is optimal for all values $k \in K$:

$$a^*(K) = \begin{cases} a_0 & \text{if } r(K) > 1 \\ a_1 & \text{if } r(K) < 1 \end{cases}$$

Else, withhold a decision.

11. An Alternative?

Kruschke [e.g. 2015, 2018] proposes the HDI+ROPE decision rule:

Calculate the 95\% highest density interval (HDI) from the posterior density $\pi(\theta|x)$.

- If HDI is completely within the ROPE, then choose $a_0$.
- If HDI is completely outside the ROPE, then choose $a_1$.
- Else, withhold a decision.

12. Comparison

<table>
<thead>
<tr>
<th>Ideal Solution</th>
<th>Imprecise ROPE-based Solution</th>
<th>HDI+ROPE decision rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>full posterior information</td>
<td>full posterior information</td>
<td>only HDI</td>
</tr>
<tr>
<td>true loss function</td>
<td>simplified loss function</td>
<td>no loss function based on $a_0$ and $a_1$</td>
</tr>
<tr>
<td>all information about loss</td>
<td>ROPE + loss magnitude as available</td>
<td>only ROPE</td>
</tr>
<tr>
<td>not arbitrary</td>
<td>not arbitrary</td>
<td>arbitrary HDI confidence level (95%)</td>
</tr>
</tbody>
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