Decision making with lower previsions

Let $\Omega$ denote the set of states of nature, and let $\mathcal{L}$ denote the set of all gambles (i.e., bounded real-valued functions) on $\Omega$. A lower prevision $\mathcal{P}$ is a lower prevision function on domain $\mathcal{L}$, where $\mathcal{L}$ is the domain of $\mathcal{P}$. The conjugate upper prevision $\mathcal{P}^U$ on domain $\mathcal{L}^U = \{ -f : f \in \mathcal{L} \}$ is defined by $\mathcal{P}^U(f) = -\mathcal{P}(-f)$. We can extend $\mathcal{P}$ to the set of all gambles $\mathcal{L}$ via its natural extension $\mathcal{P}^N$ which is finite. Moreover, $\mathcal{P}$ is a lower prevision, if and only if $\mathcal{L}$ avoids sure loss [3]. In the case that both $\Omega$ and $\mathcal{L}$ are finite, for any $f$, $\mathcal{P}(f)$ can be calculated by solving a linear program [4].

Strict partial orderings: for any two gambles $f$ and $g$, we say that

- $f > g$ whenever $\mathcal{P}(f - g) > 0$.
- $f \equiv g$ whenever $\mathcal{P}(f - g) = 0$.

Set of maximal gambles: let $>^\ast$ be a strict partial order on $\mathcal{L}$, and let $\mathcal{K}$ be a finite subset of $\mathcal{L}$. The set of maximal gambles in $\mathcal{K}$ with respect to $>^\ast$ is then defined by:

$$\text{opt}_{>^\ast}(\mathcal{K}) = \{ f \in \mathcal{K} : \forall g \in \mathcal{K} (g \not>^\ast f) \}.$$  
(1)

- $\text{opt}_{>^\ast}(\mathcal{K})$ the set of maximal gambles in $\mathcal{K}$.
- $\text{opt}_{\equiv}(\mathcal{K})$ the set of interval dominant gambles in $\mathcal{K}$.

Applying interval dominance: as every maximal gamble in $\mathcal{K}$ is interval dominant, we can eliminate non-maximal gambles in $\mathcal{K}$ by applying interval dominance first [9].

E-admissibility: $f$ is $\mathcal{E}$-admissible in $\mathcal{K}$ if there is a $p$ in the credal set of $\mathcal{L}$ such that

$$\forall g \in \mathcal{K}: \mathcal{E}(f,g) \geq \mathcal{E}(g,g).$$  
(2)

Note that any $\mathcal{E}$-admissible gamble $f$ in $\mathcal{K}$ is also maximal [4] §19.4.

Improving algorithms for finding maximal gambles

Improving Troffaes and Hable [9, p. 336]'s algorithm

- Troffaes and Hable [9]'s algorithm: Comparing with known maximal gambles before evaluating only maximal gambles when $j > i$.
- Our algorithm: $\mathcal{L}(f,j)$ is immediately maximal and evaluating only $\mathcal{L}(f,j)$ when $j > i$.
- Fast evaluation of natural extensions inside algorithms

Improving Jansen et al. [1]'s algorithm

- Jansen et al. [1]'s algorithm: Solving a single linear program per gamble.
- Improved Jansen et al. [1]'s algorithm: Reduce the number of constraints.
- Improved Troffaes and Hable [9]: if $f$ is not maximal, then we exclude it from all further iterations.

Results

| $|\Omega|\times|K|$ | $|\Omega|\times|K|$ |
|-------------------|-------------------|
| $|\Omega|\times|K|$ | $|\Omega|\times|K|$ |

| $|\Omega|\times|K|$ | $|\Omega|\times|K|$ |
|-------------------|-------------------|
| $|\Omega|\times|K|$ | $|\Omega|\times|K|$ |

Remarks

1. Our benchmarking approach does have severe computational limitations, due to the need to evaluate large numbers of natural extensions. However, this work could inspire the development of further benchmarking frameworks for testing algorithms for decision making.
2. Applying interval dominance to eliminate non-maximal gambles can make the problem smaller, and this benefits Jansen et al. [1], but not the other two algorithms.
3. We find that our algorithm, without using interval dominance, outperforms all other algorithms in all scenarios in our benchmarking.

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